Valencia-Calvo, Johnny and Villamizar-Jaimes, Aixa Eileen (2018). Modeling and simulation of piecewise smooth systems using events strategies. Cuaderno Activa, 10(1), 131 - 139.

Cuaderno
Activa
•REVISTA CIENTÍFICA DE LA FACULTAD DE INGENIERÍA•

Modeling and simulation of piecewise smooth systems using events strategies

Modelado y simulación de sistemas suaves por tramos utilizando esquemas basados en eventos.□

Johnny Valencia Calvo 1.[†] *Aixa Eileen Villamizar Jaimes 2.*[§]

Recibido: 30/11/2017 **Aprobado:** 21/02/2018

Resumen: En el siguiente documento, se muestra la descripción general de la estrategia para el modelado y simulación de sistemas con impactos por detección de eventos, así como la importancia de esta en otro tipo de sistemas dinámicos suaves por partes. El objetivo de los resultados de las simulaciones es analizar las características del comportamiento del sistema a lo largo del tiempo para un rango de velocidad diferente en un sistema de leva seguidor con impactos, como un ejemplo representativo de dinámicas discontinuas. Además, se muestra un ejemplo de un modelo apropiado del sistema leva seguidor, simulado con la estrategia de algorítmica propuesta. En este artículo, se expone una visión general del modelo suave por tramos y la simulación de un sistema leva seguidor con impactos, que se caracteriza por un perfil de leva continuo con derivada continua, para leva clásica y, un seguidor modelado como péndulo de barra. El modelo incluye tres modos dinámicos principales: modo de cuerpo libre, modo deslizante y modo de impacto. Para ello, se mostrará la descripción del algoritmo que permita simular sistemas suaves por tramos como lo son los sistemas leva seguidor con impactos. El algoritmo se desarrolló bajo una estrategia basada en detección de eventos y se implementó en Matlab. Desde el punto de vista de la teoría clásica de los sistemas dinámicos y de manera más específica de las bifurcaciones, los resultados presentados en este documento son una herramienta útil para obtener un mejor modelo de los sistemas de interés e incluso diseñar nuevas estrategias de control.

Palabras clave: sistemas dinámicos, sistema suave por tramos, sistema leva seguidor, discontinuidad, comportamientos complejos, dinámica no lineal, modelado y simulación, bifurcaciones, caos.

□ *Artículo resultado de investigación*

† *Ph.D. en Ingeniería. Tecnológico de Antioquia –IU. Medellín- Antioquia. johnny.valencia@tdea.edu.co*
§ *Master en Diseño y Gestión de Proyectos Tecnológicos. Tecnológico de Antioquia –IU. Medellín- Antioquia. aixa.villamizar@tdea.edu.co*

Abstract: In the following document, we show a general description of the strategy for the impacting system simulation by events detection and the importance of this one in another kind of piecewise-smooth dynamical systems. The goal of simulations results is to analyze the features of time behavior in a range of velocity for different models of the follower and a cam profile in a cam follower impacting system as a representative example. Furthermore, there is an example of appropriately model of cam follower systems with the algorithm strategy. In this paper, we present an overview of the piecewise smooth model and simulation of a cam-follower impacting system, characterized by a continuous cam profile with continuous derivate for classical cam and a follower modeled as a rod pendulum. The model includes three main dynamical modes, free body mode, sliding mode and impacting mode. We will show the description of the algorithm to simulate cam-follower impacting systems. The algorithm was developed under an event-driven strategy and implemented in Matlab. From the bifurcation theory point of view, the results presented in this paper can be a useful tool to obtain a better model of the systems of interest, design novel control strategies aimed to control the bifurcation diagrams.

Keywords: Piecewise smooth dynamical systems, cam-follower impacting system, modeling and simulation methodologies, dynamic simulation, bifurcation, chaos.

Introduction

The interest in piecewise smooth dynamical systems has increased last years as reported in the literature (Angulo et al., 2012; Colombo, di Bernardo, Hogan, & Jeffrey, 2012; Di Bernardo, Nordmark, & Olivar, 2008; Valencia & Osorio, 2011). There are many real applications that we can model under this framework. Typical examples are non-smooth dynamical systems, electronic switching circuits, and hybrid controllers. The non-smooth behavior usually depends on control actions, impacts or switching (state jumps). They exhibit some nonlinear phenomena similar to smooth systems but additionally, there are some strictly depending on their non-smooth nature.

These phenomena have been studied in detail and consistent theory for their classification has been derived by a number of authors. To study these phenomena, there are some analytical, numerical and experimental tools, developed under different modeling structures. As shown in (Alzate, di Bernardo, Montanaro, & Santini, 2007; Gil-Vera, 2015; Osorio, di Bernardo, & Santini, 2008; Osorio Londoño, 2007). The simulation of non-smooth systems is more complex than the linear and smooth systems due to the dramatic changes in their qualitative behavior such as jumps, impacts or discontinuity-induced bifurcations (Angulo et al., 2012; Valencia-Calvo & Osorio, 2012). Cam follower devices are an important class of impacting systems used a wide range of applications.

The most common example is the valve train of the internal combustion engine, figure 1, where the cam rotational speed provides the forcing to operate the follower. The cam is designed to rotate at a constant velocity. In practice, the velocity is varied by unwanted fluctuations and noise. Therefore, above a critical rotational speed, the follower detaches from the cam and then impacts occur.

ISSN: 2027-8101. Enero-diciembre 2018 • página 133 de 160

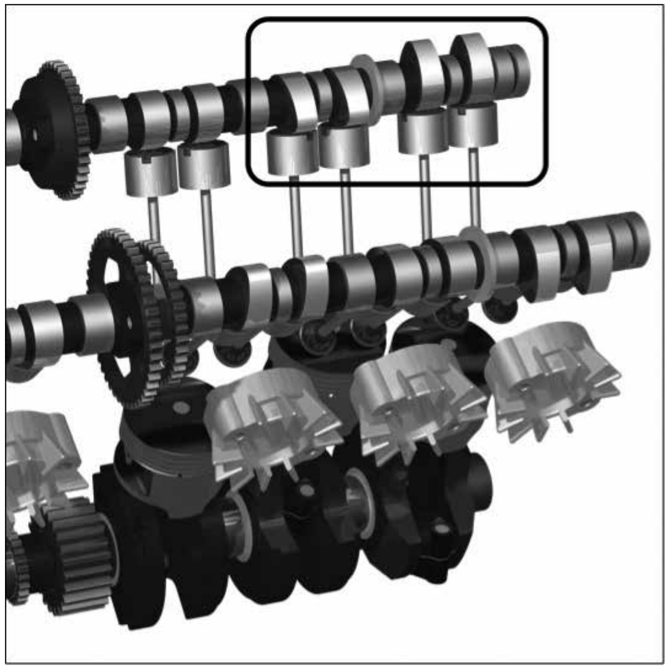

Figure 1. Internal Combustion Engine, in the dark square a cam follower system example. Source: Retrieved from (Olafpictures, 2017)

The cam profile and the cam rotational speed are the most important parameters. It has been observed that under variation of the cam rotational speed, the cam follower impacting system can exhibit complex behavior including chattering, periodic orbits, bifurcations, the coexistence of solutions until chaos solutions (Ardila Marin, Maria Isabel; Martinez Nieto, Wilson & Olmos Villalba, 2015; Cavalieri & Cardona, 2018; Osorio et al., 2008; Torabi, Akbarzadeh, Salimpour, & Khonsari, 2018).

In this context, the study of cam follower systems is used as a complement to understanding the possible phenomena that these systems can exhibit and also to implement novel control strategies based on bifurcations theory or how to design a proper cam profile to keep the cam follower in permanent contact.

In (Alzate et al., 2007; Angulo et al., 2012; Di Bernardo, Fossas, Olivar, & Vasca, 1997; Di Bernardo et al., 2008; Osorio et al., 2008) the dynamics of a cam-follower system is analyzed within the context of the bifurcation theory developed for impacting mechanical systems and is also shown that numerical simulations, experimental results and bifurcation analysis based on a simple representative model can be a fundamental tool to understand and characterize the system. In this report, a simulation tool with the strategy by events to observe the behavior of different cam-follower models, under the same algorithm. You can choose the equations and understand what kind of model is more appropriate than other to describe each system.

Events Strategy: a simulation methodology

There are different numerical strategies useful to simulate impacting systems, as shown in (Osorio et al., 2008), based on events and step fixed time. Events scheme is inspired by hybrid formulation while step fixed time is inspired by problem solutions whit complementary variables. For events scheme, there are three main states that describe system behavior, in the following section show a short description of every state and in the algorithm description the necessary conditions for changing the state.

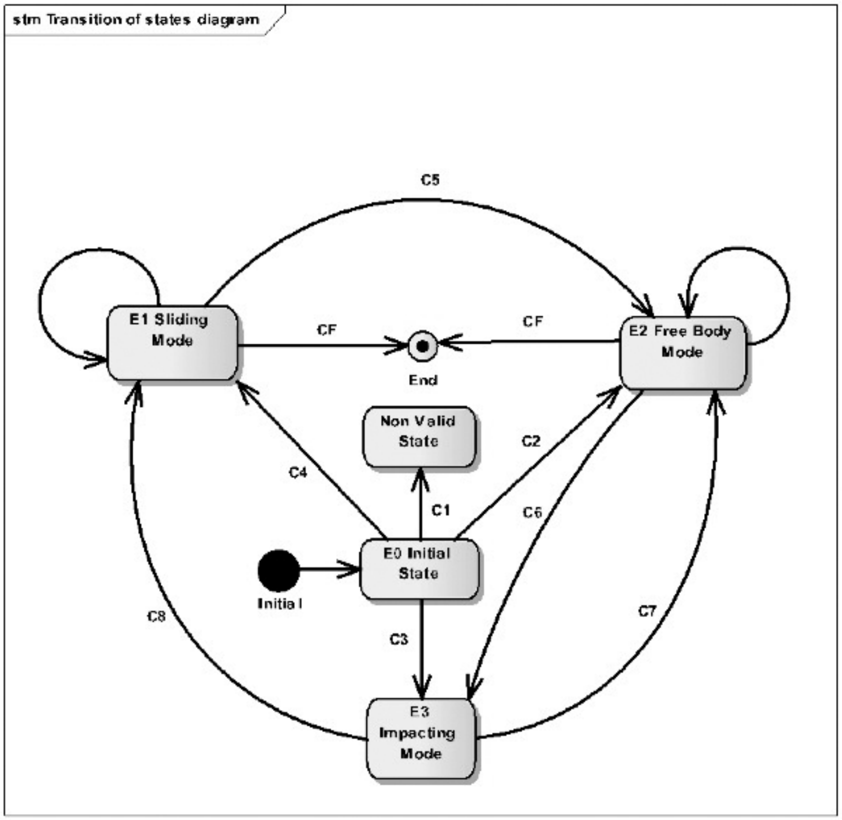

Figure 2. The transition of states diagram. Source: The authors (2017)

Algorithm Description

The goal of this algorithm is that somebody can be able to understand piecewise-smooth dynamical systems in special cam-follower impacting systems and simulate with any computational tool this kind of systems, in the figure 2 we show the machine state of this algorithm and the general description to implement the strategy.

For events framework there are three main states, we have mentioned before. We report the description of the algorithm and the necessary conditions to simulate the system. In figure 2, we build a transition of states diagram (TSD).

In the beginning, we calculated the relative acceleration and the initial conditions in order to choose which state will be the first after E0.

• When the system is in E0 the conditions to change to another state are:

1. If H(x)<0 change state to NonValid Initial State, C1=True.
2. If H(x)=0, v(x)=0 and a(x)<0 change state to E1 (Sliding Mode)}, C4=True.
3. If H(x)=0, v(x)>0 change state to E2 (Free Body Mode), C2=True.
   If H(x)=0, v(x)=0 and a(x)>0 change state to E2 (Free Body Mode), C2=True.
   If H(x)>0 change state to E2 (Free Body Mode), C2=True.
4. If H(x)=0, v(x)<0 change state to E3 (Impacting Mode), C3=True.

• When the system is in E1 the conditions to change to another state are:

1. While a(x)<0 continue in E1(Sliding Mode).
2. If a(x)>0 change state to E2 (Free Body Mode), C5=True.
3. If t>=Tfinal change state to End, CF=True. Tfinal is the limit time for the simulation.

ISSN: 2027-8101. Enero-diciembre 2018 • página 135 de 160

• When the system is in E2 the conditions to change to another state are:

1. While H(x)>0 continue in E2 (Free Body Mode).
2. If H(x)<=0 change state to E3 (Impacting Mode), C6=True.
3. If t>=Tfinal change state to End, CF=True. Tfinal is the limit time for the simulation.

• When the system is in E3 the conditions to change to other state are:

1. If v^+ (x)<=ε change state to E1 (Sliding Mode), C8=True.
2. If v^+ (x)<=ε change state to E2 (Free Body Mode), C7=True.

Example

In the following section is shown as an example of the algorithm implementation, modeling the system shown in figure 3.

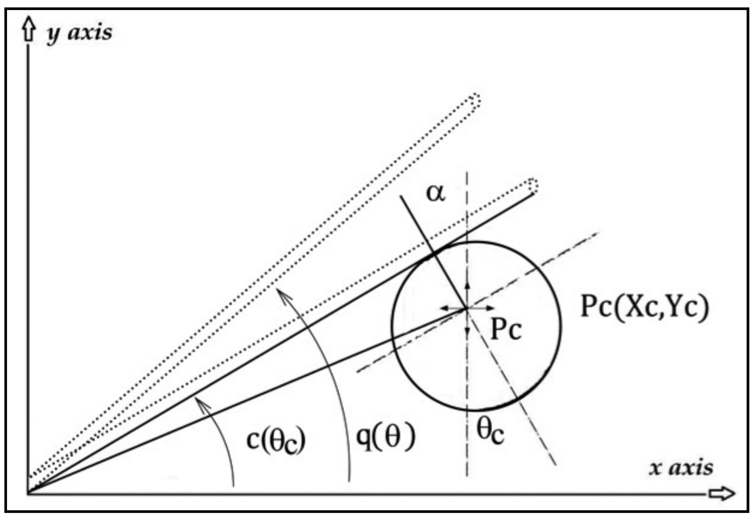

Figure 3. Cam Follower Impacting System Schematic. Source: The authors (2017)

In figure 3, we present a cam follower impacts system. In this model, an eccentric circular cam and a follower were modeled as a pendulum. To perform the simulations presented in the next section, we took into account the relative position of the pendulum with respect to the angle of rotation of the cam. Using the model described above, we can see different behaviors below which can exhibit the follower cam system with impacts.

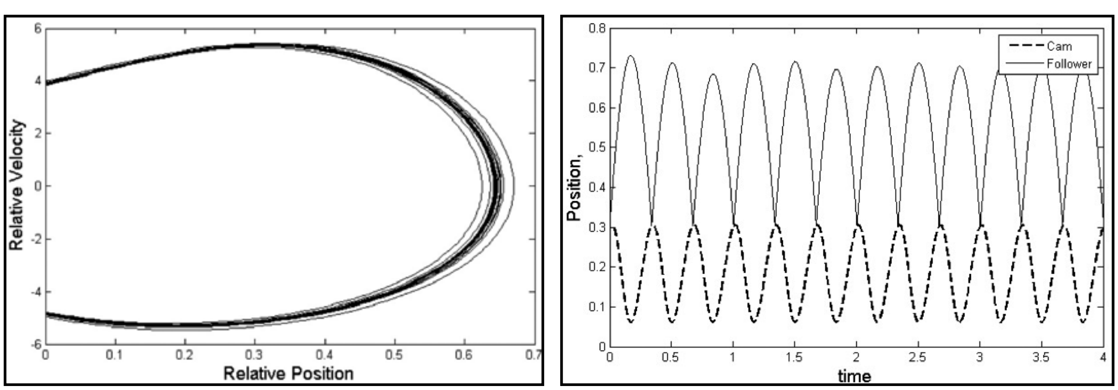

Figure 4. 1T – Periodic Behavior. Source: The authors (2017)

A fundamental characteristic of follower cam systems with impacts is the existence of different attractors for the same range of parameters. Particularly, in this case of study, there are two attractors for a series of rotational speed values with ω = [120 140] rmp. As shown in the following figures, for ω = 120 rpm there are two solutions. Depending on the initial conditions, the evolution of the system is directed attractor of the periodic solution Figure 4, to the chaotic attractor Figure 6.

The simultaneous existence of different attractors causes the behavior of the system depends strictly on the initial conditions. In this sense, the domain of Attraction and a bifurcation diagrams are a tool very useful that allow us to characterize the coexistence of solutions for equal values of the parameters in the system, and let us better understanding of their complex behavior, see more on (Taborda, Santini, Di Bernardo, & Angulo, 2009).

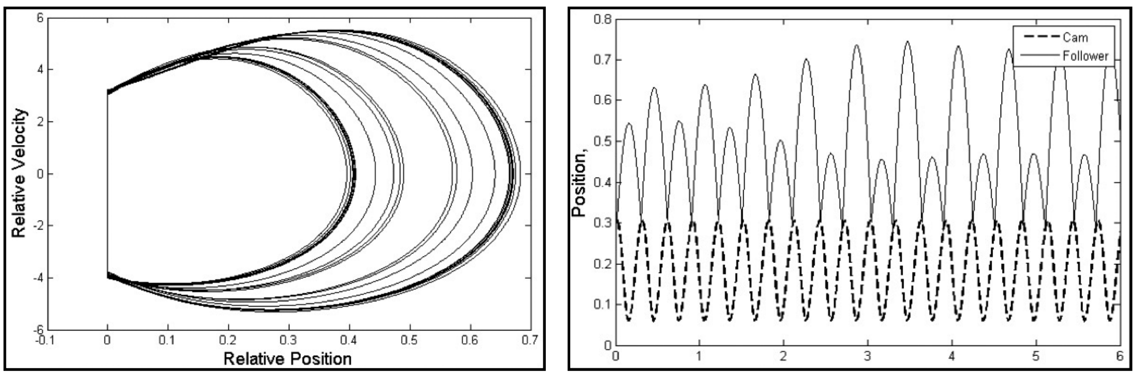

Figure 5. 2T- Periodic Behavior. Source: The authors (2017)

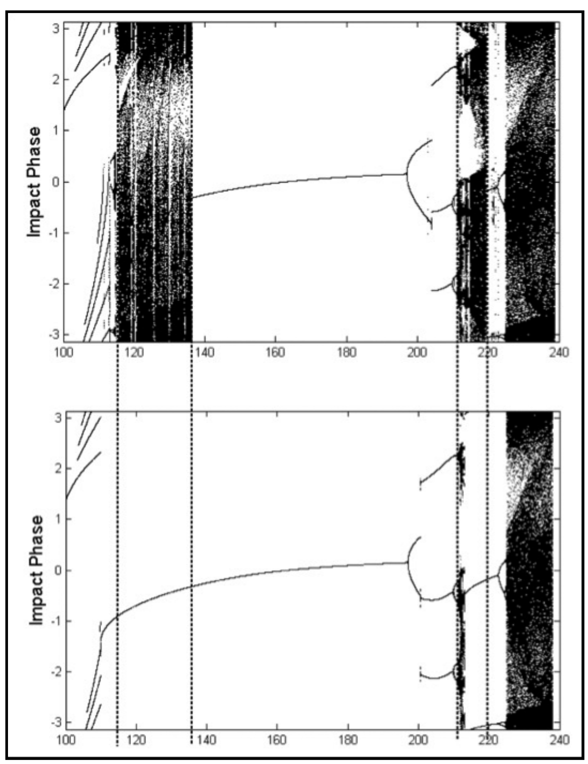

Figure 6. Bifurcation diagrams and coexistence of solutions Source: The authors (2017)

ISSN: 2027-8101. Enero-diciembre 2018 • página 137 de 160

We build a bifurcation diagram under variation of the cam rotation speed for values of =(105,210) rpm. We draw the phase of impact every period. The most relevant phenomenon is the non-smooth impact bifurcation and it is evidenced in the sudden transition between a 1T-periodic orbit, around 135 rpm, to chaos solution due to the impact of the follower with the cam profile. Other phenomena are the period of doubling, that occur around 198 rpm, this one is a smooth bifurcation. Figure 6.

A fundamental characteristic of the cam follower impacting systems is the coexistence of different attractors. Particularly, in the example shown in figure 3, it has two attractors for a range of cam rotation speed. As shown in figure 8 above, for 120 rpm there are two solutions. Depending on the initial conditions, the system evolution goes to the periodic attractor, or to the chaotic attractor. The simultaneous existence of different attractors makes the system behavior depends on the initial condition. The domains of attraction are a useful tool to characterize the coexistence of different solutions for the same values of the parameters in the system. To explore how to apply this tool, it is possible to check the results shown in (Sundar, Dreyer, & Singh, 2016; Valencia J. Osorio, 2012).

Finally, one of the solutions that the cam follower impacting system can exhibit is the chattering - accumulation of impacts - when the cam rotational speed increases and the detachment between the cam and follower occur. The accumulation of impacts determined the moment to return to sliding mode. Figure 7 shows the relative velocity of the system where is evident the accumulation point due to infinite impacts to a finite space of time. For more detailed information about the chattering phenomena see (Dankowicz & Fotsch, 2017).

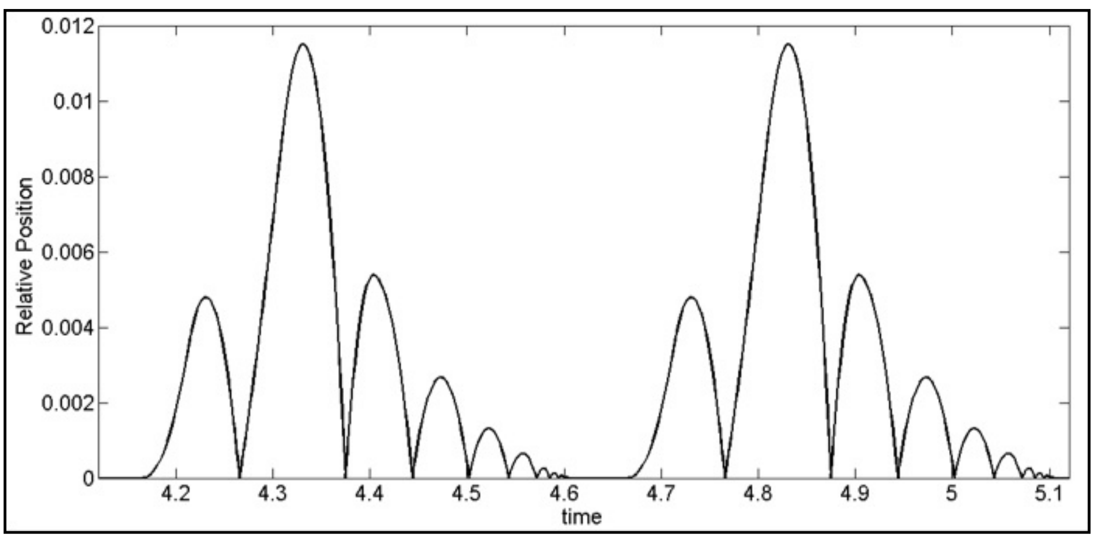

Figure 7. Chattering phenomena. Source: The authors (2017)

Conclusions

We have presented the numerical scheme to simulate piece wise smooth dynamical systems. Also, we have shown exhaustive numerical simulations for characterizing all phenomena that this kind of systems can exhibit. Additionally, our proposal is generalized the scheme under we should simulate impacting or non-smooth dynamical systems. To simulate this system is necessary to know the equations that describe its flow in every state and the conditions in the boundaries to transition between the dynamical states.

Whit this information and the transition states diagram, it is possible to simulate a wide range of piece-wise smooth dynamical systems.
From the bifurcation theory point of view, the results presented in this paper can be a useful tool to obtain a better model of the systems of interest, design novel control strategies aimed to control the bifurcation diagrams.

We choose an adequate framework to model and simulate the systems. With the algorithm implementation, we proved that a single degree of freedom is enough to represent the majority of phenomena that the system can exhibit. The cam rotation speed is a very important parameter of bifurcation as evidenced in the bifurcation diagram. Additionally, cams with restrictions in their derivative as discontinuity points can induce novel bifurcations or sudden transitions to nonlinear behavior that can damage the mechanical rig.

A future work of great importance is the experimental verification of the different dynamics that were presented in this work. With these results and the analytical tool proposed, we can count on a better understanding of the complex dynamics of the follower cam system and, in general terms, the complex behavior of piecewise smooth systems. As mentioned before, in the literature it is modeled, analysis and simulation of some systems with impacts but not much information about assemblies experimental that allow to verify their dynamics.

References

Alzate, R., di Bernardo, M., Montanaro, U., & Santini, S. (2007). Experimental and numerical verification of bifurcations and chaos in cam-follower impacting systems. Nonlinear Dynamics, 50(3), 409. https://doi.org/10.1007/s11071-006-9188-8

Angulo, F., Olivar, G., Osorio, G. A., Escobar, C. M., Ferreira, J. D., & Redondo, J. M. (2012). Bifurcations of non-smooth systems. Communications in Nonlinear Science and Numerical Simulation, 17(12), 4683–4689. https://doi.org/10.1016/j.cnsns.2011.07.021

Ardila Marin, Maria Isabel; Martinez Nieto, Wilson & Olmos Villalba, L. C. (2015). Outsourcing de mantenimiento , una alternativa de gestión de activos en el sector productivo de bienes y servicios . Cuaderno Activa, 123–133.

Cavalieri, F. J., & Cardona, A. (2018). Non-smooth model of a frictionless and dry three-dimensional revolute joint with clearance for multibody system dynamics. Mechanism and Machine Theory, 121, 335–354. https://doi.org/10.1016/j.mechmachtheory.2017.09.018

Colombo, A., di Bernardo, M., Hogan, S. J., & Jeffrey, M. R. (2012). Bifurcations of piecewise smooth flows: Perspectives, methodologies and open problems. Physica D: Nonlinear Phenomena, 241(22), 1845–1860. https://doi.org/10.1016/j.physd.2011.09.017

Dankowicz, H., & Fotsch, E. (2017). On the Analysis of Chatter in Mechanical Systems with Impacts. Procedia IUTAM, 20, 18–25. https://doi.org/10.1016/j.piutam.2017.03.004

Di Bernardo, M., Fossas, E., Olivar, G., & Vasca, F. (1997). Secondary Bifurcations and High Periodic Orbits in Voltage Controlled Buck Converter. International Journal of Bifurcation and Chaos, 7(12), 2755–2771. https://doi.org/10.1142/S0218127497001862

ISSN: 2027-8101. Enero-diciembre 2018 • página 139 de 160

Di Bernardo, M., Nordmark, A., & Olivar, G. (2008). Discontinuity-induced bifurcations of equilibria in piecewise-smooth and impacting dynamical systems. Physica D: Nonlinear Phenomena, 237(1), 119–136. https://doi.org/10.1016/j.physd.2007.08.008

Gil-Vera, V. D. (2015). Forecasting Electricity Demand for Small Colombian Populations . Cuderno Activa, 7(1), 111–119.

Olafpictures. (2017). motocicleta-motor-raytracing-hacer-2037242. Retrieved from https://pixabay.com/es/motocicleta-motor-raytracing-hacer-2037242/

Osorio, G., di Bernardo, M., & Santini, S. (2008). Corner-Impact Bifurcations: A Novel Class of Discontinuity-Induced Bifurcations in Cam-Follower Systems. SIAM Journal on Applied Dynamical Systems, 7(1), 18–38. https://doi.org/10.1137/060666433

Osorio Londoño, G. A. (2007). Complex Behavior in Impacting Systems. University of Naples Federico II.

Sundar, S., Dreyer, J. T., & Singh, R. (2016). Estimation of impact damping parameters for a cam-follower system based on measurements and analytical model. Mechanical Systems and Signal Processing, 81, 294–307. https://doi.org/10.1016/j.ymssp.2016.02.033

Taborda, J. A., Santini, S., Di Bernardo, M., & Angulo, F. (2009). Active chaos control of a cam-follower impacting system using FPIC Technique. IFAC Proceedings Volumes (IFAC-PapersOnline) (Vol. 2). IFAC. https://doi.org/10.3182/20090622-3-UK-3004.00061

Torabi, A., Akbarzadeh, S., Salimpour, M., & Khonsari, M. M. (2018). On the running-in behavior of cam-follower mechanism. Tribology International, 118(September 2017), 301–313. https://doi.org/10.1016/j.triboint.2017.09.034

Valencia-Calvo, J., & Osorio, G. A. (2012). Stability analysis of cam follower impacting system. IEEE Xplorer. https://doi.org/978-1-4614-6/12/$31.00

Valencia, J., & Osorio, G. (2011). Nonlinear numerical analysis of a cam-follower impacting system. In 2011 IEEE Second Latin American Symposium on Circuits and Systems (LASCAS) (pp. 1–4). Ieee. https://doi.org/10.1109/LASCAS.2011.5750317

Valencia J. Osorio, G. (2012). Bifurcación por impacto con esquina en el sistema leva seguidor. Universidad Nacional de Colombia. Retrieved from http://www.bdigital.unal.edu.co/6866/